A Nonlinear Elastic Class of Materials

P. A. Johnson (paj@lanl.gov), J. N. TenCate, T. Shankland, and D. E. Smith (EES-11), T. Claytor and D. Summa (ESA-MT), T. Darling (MST-10), A. Migliori (MST-NHMFL)

It is becoming clear that numerous materials (sand, soil, some ceramics, concrete, etc.) share the elastic properties of rock. These materials have one or more of the following properties: strong nonlinearity, hysteresis in stress-strain relation, and discrete memory. Primarily, it is compliance, the microscopic linkages between the rigid components in the materials, that gives these materials their unusual elastic properties, termed "nonlinear mesoscopic elasticity." Our studies on these still-unexplained properties of certain materials have important implications for basic and applied research, including nondestructive testing of materials (including weapons components), studying earthquake strong-ground motion, and evaluating manufacturing processes such as assembly lines or concrete manufacturing.

Materials with nonlinear microscopic elasticity stand in contrast to liquids and crystalline solids, whose elasticity is due to contributions of atomic-level forces (i.e., materials with atomic elasticity). Atomic-elastic materials are well described by the traditional (Landau) theory of elasticity, and mesoscopic materials are well described by the P-M (Preisach-Mayergoyz) space model of nonlinear elasticity developed by Guyer and McCall. However, the nonlinear mechanisms in mesoscopic elastic materials still remain a mystery.

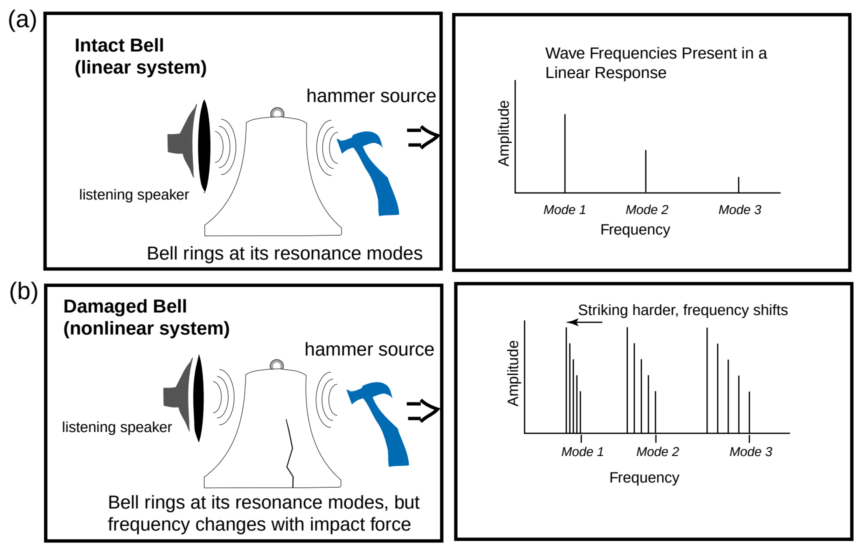

Figure 1. Wave Resonance Behavior in a Bell.
(a) Striking a bell of normal material excites the resonance modes whose frequencies are shown to the right. (b) Striking a bell that has a very small crack (or one composed of mesoscopic nonlinear material) excites modal frequencies whose values depend on how hard we strike (bottom right). This result is a nonlinear effect—a change in wave frequency with wave amplitude.

Introduction

Strike a normal bell, and the bell rings at its resonance modes. Strike it harder, and the bell rings at the same tones, only louder (Figure 1a). Now imagine a bell composed of granite, compressed powdered metal, or other mesoscopic nonlinear material. We strike the bell gently, and it rings normally. Striking it harder, we find, to our surprise, that the tone drops in frequency ever so slightly. Striking it even harder, the tone drops further. The frequency shift is a manifestation of nonlinearity resulting from certain properties of these and other materials. Interestingly, a crack in the bell, even a small crack, will make the bell respond nonlinearly as well—that is, like a rock, etc. (Figure 1b). That remarkable property can be applied to detect damage in materials using a nondestructive technique we call nonlinear resonant ultrasound spectroscopy (NRUS). The NRUS method is being patented.

There are additional manifestations of nonlinearity. If a normal bell is excited with an audio speaker using arbitrarily chosen input frequencies of 440 and 8000 cycles per second (Hz), the bell rings at the two input frequencies (Figure 2a). If we do the same thing with a cracked bell (or one made of granite, powdered aluminum, etc.), interesting things happen again (Figure 2b). Not only does the bell ring at 440 and 8000 Hz, but other frequencies abound. We can detect two, three, and four times each input frequency (880, 1,320, and 1,740 Hz, and 16,000, 24,000, and 32,000 Hz, respectively) and also the sum and difference frequencies (8,000 ± 440 Hz), which are called sidebands. We call this technique nonlinear wave modulation spectroscopy (NWMS), which is also being patented.

Both the amount of the resonance-peak change with amplitude (Figure 1b) and the appearance of large numbers of new frequencies inside the material (Figure 2b) are not expected results! They are the result of nonlinear interaction of the sound in the damaged bell.

What causes these behaviors? The common theme is the presence of soft regions contained within harder regions. For instance, in a rock, the soft regions are the grain contacts and microcracks; in a crystal, they are dislocations; in powdered compressed metals, they are hard sintered components glued together with softer material much like a rock. A crack in a piece of metal acts as a localized soft region in an otherwise stiff material. Figure 3 illustrates some of these features in materials.

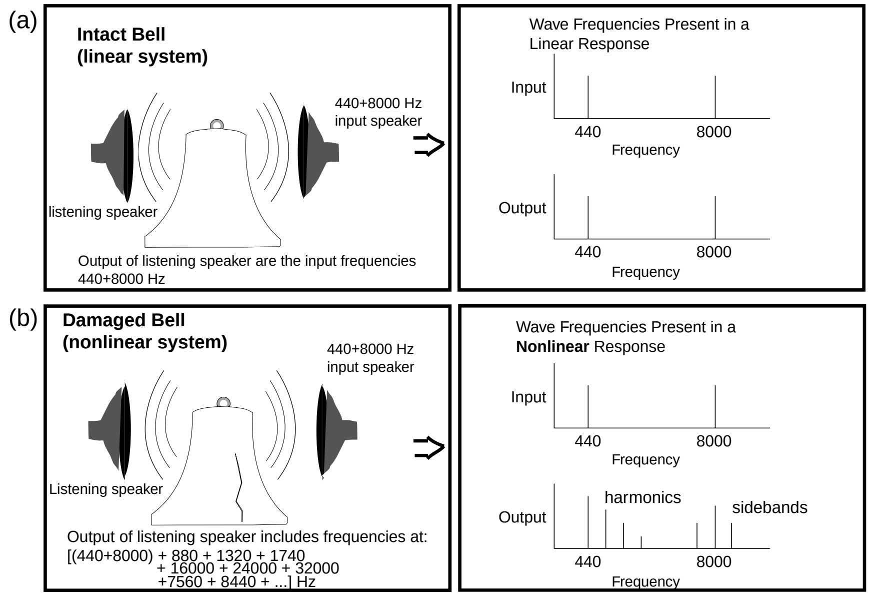

Figure 2. Wave Harmonics and Modulation in a Bell.
(a) A normal bell responds to two frequencies from a speaker with those same two frequencies. (b) For a cracked bell, nonlinear mixing (multiplication) occurs. The two frequencies multiply with themselves to create harmonics and with each other to create sum and difference frequencies (sidebands). We call this nonlinear wave modulation spectroscopy (NWMS). It is being patented.

The Fundamental Measure of Nonlinearity: Stress-Strain

Suffice it to say that the presence of the soft features inside otherwise hard materials gives rise to all forms of elastic nonlinear behavior. Elasticity? That is simply Hooke's law, which relates the applied stress to the resulting deformation, or strain, via a quantity called the modulus. Compressional modulus and shear modulus are among the "spring constants" of linear elasticity theory. Stress is a tensor quantity with units of force per area that, in some instances, coincides with pressure. Stress is usually measured in pascals (0.1 MPa = 1 atm).

A cylinder of Pyrex glass and a block of copper are well described by linear elasticity. They are what we call atomic elastic materials. Not so the materials above. Their elasticity is strikingly nonlinear, is hysteretic, and has discrete memory. For instance, if we study the static behavior of any of the above materials, we will see nonlinearity in the relationship of applied static stress versus the responding deformation. Figure 4 compares the stress-strain relationships of a linear material, Pyrex glass, and a nonlinear material, Berea sandstone.

All or some of these properties (nonlinear/hysteretic/discrete memory) are seen in the elasticity of ceramics, cement, and concrete materials that we think of as consolidated materials (i.e., materials whose primary properties are due to the process of their construction or consolidation). Put another way, the elastic properties of a sandstone and similar materials are not at all those of the grains but rather are conferred by the bonds between the grains (typical scale 1 μm), with the grains acting as essentially rigid elements. It is the bond system, the set of effective elastic elements between the grains and the cracks within the grains themselves, that control the behavior of the elastic properties. These bonds are mesoscopic in size, which is why we call such materials nonlinear mesoscopic elastic materials.

Nonlinearity and Acoustic Waves in Damaged Materials

Returning to the subject of cracked material, we have shown that the nonlinearity resulting from the presence of cracks is an extremely sensitive indicator of damage. The undamaged portion of the sample produces nearly zero nonlinear effects. The damaged portion acts as a nonlinear mixer (multiplier). It is a localized effect. We can easily tell the difference between an undamaged and a damaged object using a frequency spectrum analysis. In fact, we are not aware of a more sensitive, more rapid, and easier-to-apply method for detecting and examining material damage.

Our studies have shown that the nonlinear response of a sample provides a quick, qualitative test of pass/fail (go/no-go) in numerous metal components such as alternator housings, engine bearing caps, gears, Plexiglas, synthetic slates, weapons components, etc., where damage is localized. The elastic nonlinear response is also useful in examining

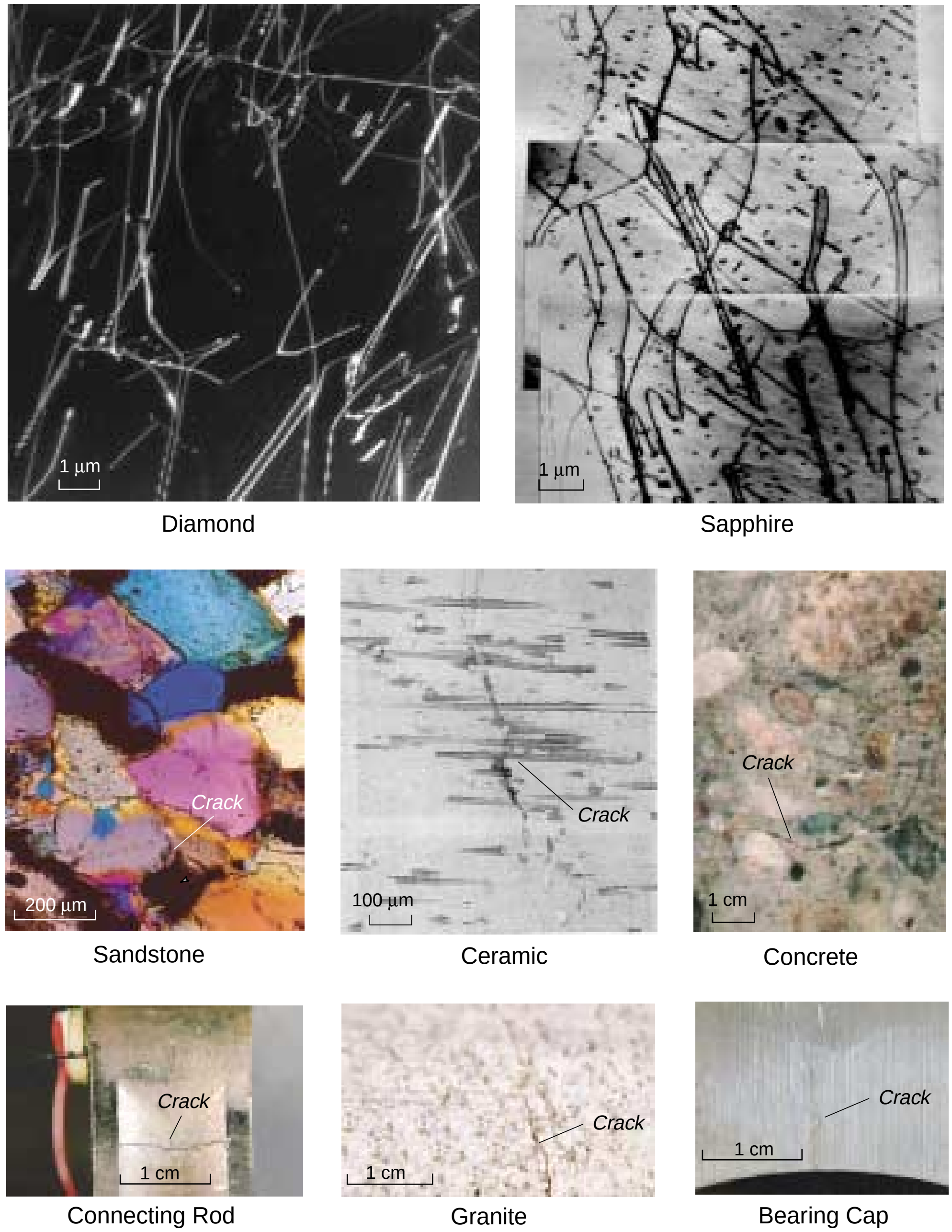

Figure 3. Features that Provide a Nonlinear Response.
Dislocations in type-2 diamond and sapphire, the soft bond system in sandstone, a single crack in ceramic (barium magnesium silicate doped with borosilicate glass), and cracks in concrete, a connecting rod, granite, or a damaged bearing cap are all physical features that can result in a nonlinear response.

the physical state of volumetrically damaged materials such as concrete, rock core, and other porous materials (including the effects of fluid saturation) and is being applied to characterize dislocations in metals and to study progressive damage in these materials (Figure 5).

Summarizing, we can say that as a material fatigues or is damaged, dislocations, cracks, and flaws may be introduced, resulting in a significant change in the material's nonlinear elastic wave behavior. This behavior is manifest in two primary manners when sound is applied to the object. First, under resonance conditions (such as the bell), the resonance tone changes as the applied volume is increased. Second, under resonance, continuous-wave, or pulse-wave excitation, frequency-mixing spectral components, such as wave harmonics, appear. These effects are enormous in damaged material but nearly unmeasureable in undamaged materials. They are the signatures of damage. Linear methods in acoustical nondestructive testing rely on either reflected wave energy from a crack, wave speed changes, or amplitude changes. None of these linear wave characteristics are as sensitive as the nonlinear response of the material.

In volumetrically damaged materials, microfeatures such as dislocations are responsible for the nonlinear behavior. It is very interesting that volumetric and local damage over several orders of magnitude in scale (~10^{-9} to 10^{-1}) provide very similar nonlinear characteristics (e.g., Guyer and Johnson, 1999)! That is, there are close similarities between the nonlinear response from the presence of dislocations in a sample and a single macrocrack in a sample. The similarities are currently under intense scrutiny to determine why this is so. Dislocations, soft grain contacts in rock and concrete, microcracks, and macrocracks can all lead to a large and complex nonlinear wave response (Figure 3).

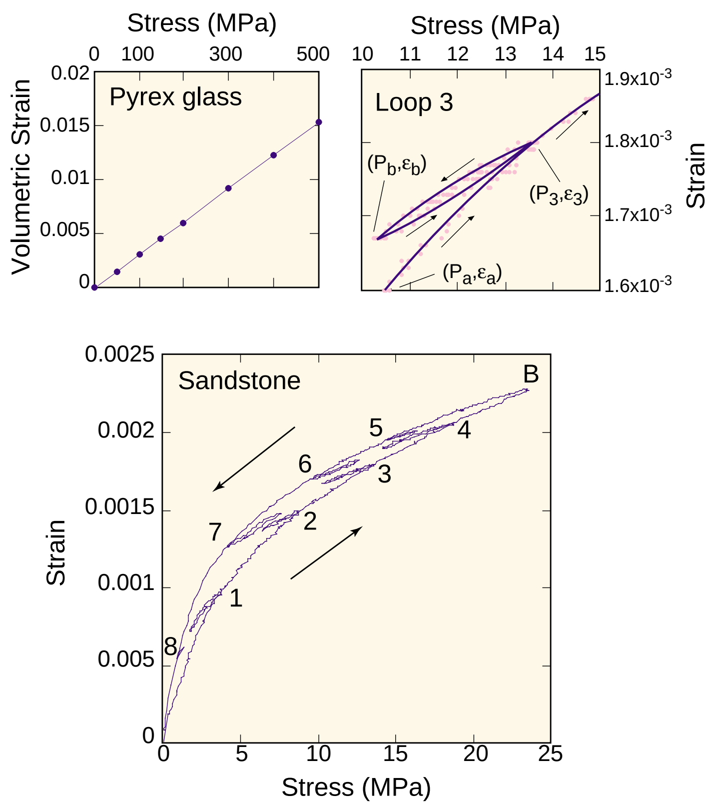

Figure 4. Stress-Strain Relationships.
Here, material is stressed by being squeezed with a piston, and the resulting deformation, or strain, is measured. Pyrex glass (top left), a normal or atomic elastic material, has a stress-strain relationship that is linear from 0 to 500 MPa. On the other hand, Berea sandstone (bottom), a mesoscopic elastic material, is highly nonlinear as evident in the curved response from 0 to 25 MPa. The sandstone also shows hysteresis—when stress is reversed at B, strain only partially reverses, and a loop forms. The small numbered loops are the result of other small excursions in the stress; details of the third reversal are shown at top right.

Example of a Nondestructive Testing Application

As a practical example, we show NWMS experiments in automobile engine bearing caps that determine whether or not damage is present. In these tests, one high-frequency wave and several low-frequency waves were used simultaneously as input. Thus, when damage is present, we would expect mixing of all waves with each other, leading to the creation of many harmonics and sidebands. Figures 6a and 6b show the frequency wave spectrum around the sideband frequencies of the undamaged and damaged samples, respectively. The damaged sample (seen in Figure 5) contains a crack approximately several millimeters deep and 1 cm long and clearly failed the go/no-go test. Note that we observed no change in linear wave speed or wave dissipation between the two samples despite the fact that the nonlinear response is very different.

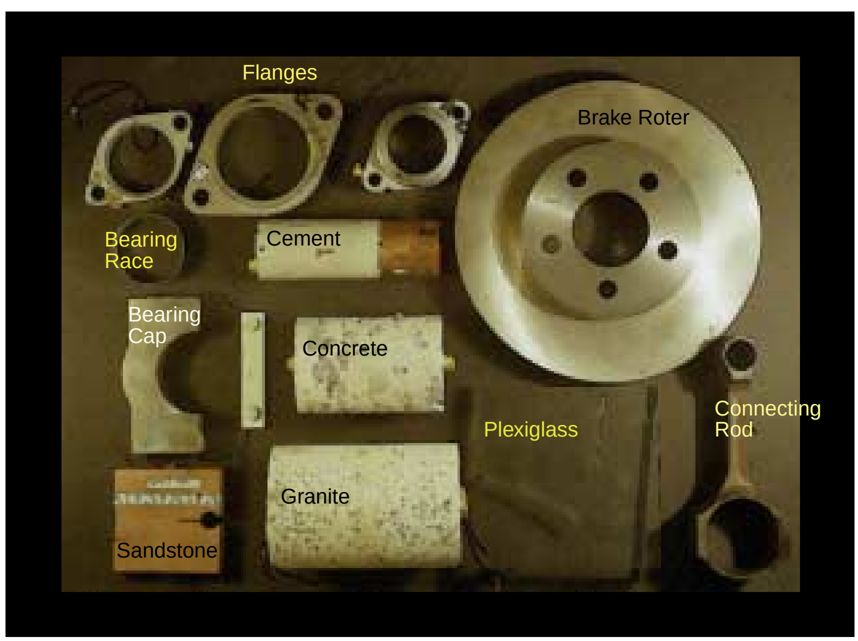

Figure 5. Objects Tested for Damage.
All of the samples shown here are damaged in some manner, either with a small localized crack or with volumetric damage.

NWMS is ideal for monitoring progressive damage in materials as well, as the experiment with the plastic rod shown in Figure 7. The rod, fixed at one end, was shaken at its fixed end in shear until failure

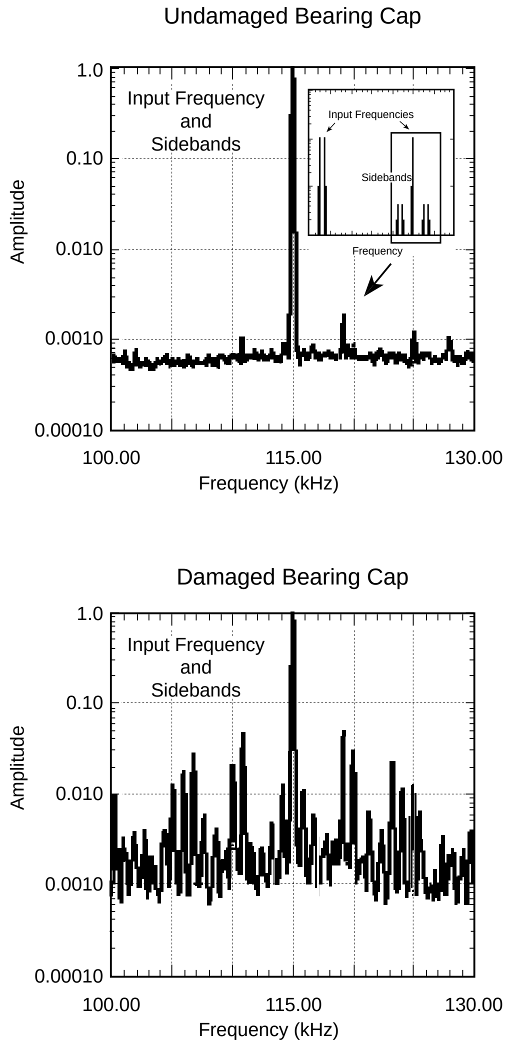

Figure 6. A Nonlinear Wave Modulation Spectroscopy Experiment.
The graphs are NWMS frequency spectra from wave modulation tests of (a) undamaged and (b) damaged engine bearing caps. The inset (top) shows a full spectrum including the sideband portion. There are many sidebands because multiple frequencies were input simultaneously in continuous-wave mode.

occurred. Linear and nonlinear behavior was monitored at each step. The linear responses were relatively insensitive to induced damage until just before failure, whereas the nonlinear response was affected early in the damage process and became enormous quickly. Nonlinear means are far superior to linear means in progressive damage detection.

Strong Ground Motion During Earthquakes

If the 1906 San Francisco earthquake took place today, the costs to replace the infrastructure would surely exceed a trillion dollars. One key to mitigating such a disaster is understanding and modeling the physical system on which structures are built (in the worst case, a structure built on soft sedimentary layers that can significantly amplify seismic waves). This process is well understood by both the seismic and civil engineering communities; unfortunately, a key aspect that is not well understood is the enormous effect of the nonlinear response of the system on structural damage and failure. In 1997, in an article in *Nature* (Field et al., 1997), we first demonstrated unequivocally that nonlinearity is a significant influence in strong ground motion.

During a large earthquake, buildings collapse and are damaged because soft sediments just beneath the Earth's surface can amplify seismic shear waves as a result of (1) seismic wave velocity gradations and (2) trapping of wave energy, creating resonances and dramatically increasing seismic wave amplitudes. This amplification is called strong ground motion. Amplified wave frequencies that correspond to the resonant modes of a structure can couple into the structure, leading to damage and ultimately, failure. The greatest unknown in this physical system is the influence of the nonlinear response, which can dramatically alter sediment resonance frequencies and wave amplitudes in ways that are only now becoming clear.

In an earthquake wave in a surface layer, every frequency component in the wavefield multiplies nonlinearly with itself and with every other frequency component, just as in the examples shown above. For instance, any two angular frequencies ω_1 and ω_2 will multiply with each other, creating additional frequency components at $\omega_1 \pm \omega_2$, $2\omega_1$, $2\omega_1 \pm \omega_2$, $3\omega_1$, $3\omega_2$, and so forth. The interaction is amplitude-dependent; for example, in the case of a resonating layer, resonance peak frequencies and mixing of these frequencies dominate the amplitude of the spectral response. As the amplitude increases, energy is progressively redistributed from the resonance frequency to other portions of the spectrum. As a result of wave mixing, we expect new frequency components to appear, especially at multiples of resonance frequencies as the driving force (the source magnitude) becomes larger and larger.

The effects of nonlinear wave mixing are important because the models most often used in prediction of strong ground motion do not incorporate frequency mixing, and strain softening is not well understood. We have unique experience and a unique

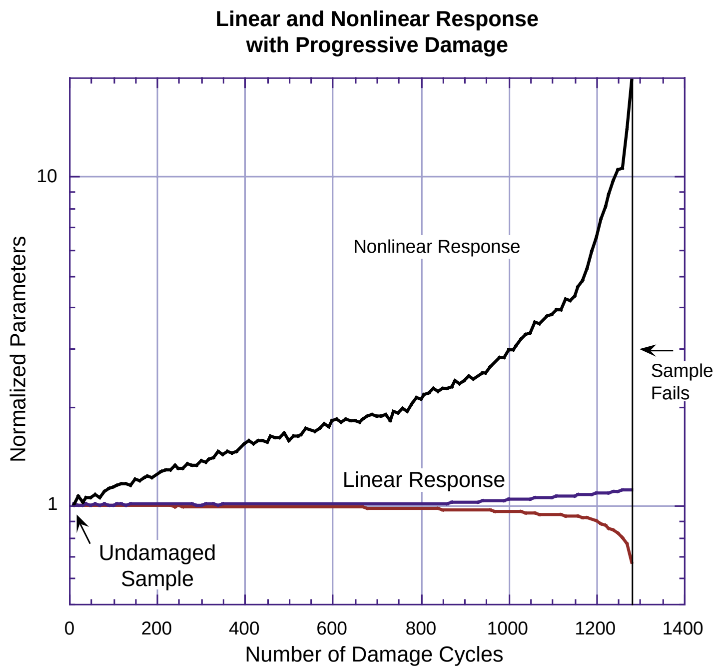

Figure 7. Progressive Damage in Plastic.
This plot (courtesy of Peter Nagy) of linear and nonlinear response as a function of damage cycles for a plastic rod shows that the linear responses (wave dissipation in blue and wave speed in red) are relatively insensitive to induced damage until just before the sample fails at nearly 1,300 cycles. However, the nonlinear response is affected early in the damage process and grows quickly.

group of analytical tools that we developed for strong-motion data analysis. For instance, we developed the method of constant strain analysis and resonance-template-matched filtering that take advantage of both phase and amplitude of the Fourier transformed seismic signal. These novel methods (currently being patented) provide us unparalleled sensitivity for studying resonance frequency changes (or jumps) as a function of earthquake magnitude and guide our modeling efforts of the nonlinear response. Furthermore, our experience in understanding nonlinear behavior in earth and damaged materials is unparalleled and provides us a unique point of view for addressing this problem.

Conclusion

Dynamic nonlinear response of mesoscopic materials is a new and extremely exciting domain of study, both from the standpoint of basic research into materials and from that of applications. We have identified a new universal class of materials that we believe includes geomaterials and many types of manufactured materials, including damaged materials. There are potentially many applications of enormous economic and safety impact that will evolve from nonlinear applications. Applications and spin-off research will affect a number of problems, from designing earthquake-resistant structures to eliminating flawed components fabricated on an assembly line and monitoring long-term aging in infrastructure. Further, application to structures after an earthquake may well provide valuable information regarding the extent of the damage. We anticipate that within 10 years, nonlinear methods may be used routinely in applications as diverse as quality control in manufacturing processes, quality control of concrete curing, monitoring reactor containment walls for damage, inspecting aircraft and spacecraft for damage, and observing fatigue damage in buildings, bridges, tunnels, and gas or oil pipe lines. ■

Acknowledgements

Funding: Technology Partnership Program, Los Alamos Institute of Geophysics and Planetary Physics, Laboratory Directed Reseach Development Program, and U.S. Department of Energy Office of Basic Energy Sciences.

Further Reading

Field, E. H., P. A. Johnson, I. A. Beresnev, and Y. H. Zeng. 1997. Nonlinear ground-motion amplification by sediments during the 1994 Northridge earthquake. *Nature* **390**: 599–602.

Guyer, R. A., and P. A. Johnson. 1999. The astonishing case of mesoscopic elastic nonlinearity. *Physics Today* **52**: 30–35.

Guyer, R. A., J. N. TenCate, and P. A. Johnson. 1999. Hysteresis and the dynamic elasticity of consolidated granular materials. *Physical Review Letters* **82**: 3280–3283.

Johnson, P. 1999. The new wave in acoustic testing. *Materials World: The Journal of Inst. Materials* **7**: 544–546.

TenCate, J. A., E. Smith, and R. A.Guyer. 2000. Universal slow dynamics in granular solids. *Physical Review Letters* **85**: 1020–1023.

Van den Abeele, K. E. A., J. Carmeliet, J. A. TenCate, and P. A. Johnson. 2000. Nonlinear elastic wave spectroscopy (NEWS) techniques to discern material damage: Part II. Single mode nonlinear resonant acoustic spectroscopy. *Research on Nondestructive Evaluation* **12**: 31–42.

Van den Abeele, K. E. A., P. A. Johnson, and A. Sutin. 2000. Nonlinear elastic wave spectroscopy (NEWS) techniques to discern material damage: Part I. Nonlinear wave modulation spectroscopy (NWMS). *Research on Nondestructive Evaluation* **12**: 17–30.

Also see Los Alamos Center for Nonlinear Elastic Materials at http://www.ees4.lanl.gov/nonlinear/.